Commutation technique for an exciton photocreated close to a metal

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Abstract

Recently, we have derived the changes in the absorption spectrum of an exciton when this exciton is photocreated close to a metal. The resolution of this problem – which has similarities with Fermi edge singularities – has been made possible by the introduction of "exciton diagrams". The validity of this procedure relied on a dreadful calculation based on standard free electron and free hole diagrams, with the semiconductor-metal interaction included at second order only, and its intuitive extention to higher orders. Using the commutation technique we recently introduced to deal with interacting excitons, we are now able to *prove* that this exciton diagram procedure is indeed valid at any order in the interaction.

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Interactions with excitons have always been a tricky problem to handle properly. The interactions being in fact interactions with free electrons and free holes, one a priori has to crack the excitons into electrons and holes, in order to really know their effects. This leads to see the exciton as the sum of ladder diagrams [1] between one electron and one hole, with possibly, once in a while, an interaction of this electron or this hole with something else. Although fully safe, this approach becomes very fast dreadfully complicated, as can be seen from the simplest problem on interacting excitons studied in reference (2), namely an exciton photocreated close to a metallic "mirror". It is indeed the simplest problem on interacting excitons, in the sense that the photocreated electron and the metal electrons are discernable (being spatially separated) so that there is no Pauli exclusion between them. This Pauli exclusion, and the exchange processes associated to the indiscernability of the carriers, is an additional, but major, difficulty for interacting exciton problems. Rather recently, we have developed a "commutation technique" [3,4] which allows to cleanly identify contributions coming from Coulomb interaction *between excitons* and contributions coming from possible exchange *between carriers*. Using this commutation technique, we can derive the correlations between excitons at any order *exactly*. We have already been able to prove that the effective bosonic hamiltonian for excitons quoted by everyone up to now cannot be correct : First, it is not even hermitian [3] ; second, it misses purely Pauli terms [3] ; third, and worse, the concept of effective hamiltonian itself has to be given up [5] because, whatever the exciton-exciton part is, it cannot reproduce the exciton correlations correctly, due to the complexity of the exchange processes. If such an effective hamiltonian were correct, exciton diagrams could obviously be used, with boson-exciton propagators and interaction vertices deduced from the interacting part of the hamiltonian. Since such an effective hamiltonian is incorrect, the validity of the exciton diagram procedure is actually not established at all.

At the time we studied the problem of an exciton photocreated close to a metal and the changes in the exciton absorption spectrum induced by the semiconductor-metal interaction, we had not yet developed this commutation technique. This is why we safely used standard diagrams [6] with free electrons and free holes and Coulomb interactions between them. We were able to put the electron-metal and hole-metal interactions at second order only. At this order, we proved that the sum of all the seven complicated

diagrams corresponding to these second order processes ends up with the same result as the one derived in an extremely simple way, by using intuitive "exciton diagrams" : In these, the exciton propagator was taken to be

$$G_x(\omega;\nu,\mathbf{Q}) = \frac{1}{\omega - E_{\nu,\mathbf{Q}} + i\eta} \ , \tag{1}$$

where $E_{\nu,\mathbf{Q}} = \varepsilon_\nu + \mathcal{E}_\mathbf{Q}$ is the energy of the $(\nu,\mathbf{Q})$ exciton, ν being the relative motion state index and $\mathbf{Q}$ the center of mass momentum. The exciton-metal vertex was somehow *cooked in a reasonable way* from the bare electron-metal and hole-metal interactions.

Since there were no hope to calculate standard electron-hole diagrams with more than two electron-metal and hole-metal interactions, we assumed that the exciton diagram procedure, which looked physically quite reasonable, should hold at any order.

By studying this problem in the light of our commutation technique, we are now able to *prove* that this exciton diagram procedure is indeed fully correct.

Let us reconsider this problem from the beginning : A highly doped 2D quantum well is set at a distance d from an empty quantum well in which an exciton is photocreated. The metal Fermi sea reacts to the sudden appearance of the photocreated electron-hole, and its change, in turn, modifies the photon absorption. Of course, similarities with Fermi edge singularities [7–9] follow from this Fermi sea reaction.

The hamiltonian of this semiconductor-metal coupled system reads $H = H_{\rm sc} + H_{\rm m} + W_{\rm sc-m}$, where $H_{\rm sc}$ is the semiconductor hamiltonian and $H_{\rm m}$ is the metal hamiltonian. The semiconductor-metal coupling $W_{\rm sc-m}$ reads

$$W_{\rm sc-m} = \sum_{\mathbf{q}\neq\mathbf{0}} \sum_{\mathbf{k}} V(\mathbf{q}) \left(a^\dagger_{\mathbf{k}+\mathbf{q}} a_\mathbf{k} - b^\dagger_{\mathbf{k}+\mathbf{q}} b_\mathbf{k} \right) w_{-\mathbf{q}} \ , \tag{2}$$

$$w_\mathbf{q} = \sum_\mathbf{p} c^\dagger_{\mathbf{p}+\mathbf{q}} c_\mathbf{p} \ , \tag{3}$$

$a^\dagger_\mathbf{k}$, $b^\dagger_\mathbf{k}$ and $c^\dagger_\mathbf{k}$ being the semiconductor electron, semiconductor hole and metal electron creation operators respectively, while, for metal and semiconductor d apart [2], $V(\mathbf{q}) = e^{-qd} 2\pi e^2/Sq$.

The absorption of a photon $(\Omega,\mathbf{Q})$, given by the Fermi golden rule, is proportional to the imaginary part of the response function

$$S(\Omega,\mathbf{Q}) = \langle i|U \frac{1}{\Omega + \mathbb{E}_0 - H + i\eta} U^\dagger |i\rangle \ , \tag{4}$$

where $|i\rangle = |v\rangle \otimes |0\rangle$, with $|v\rangle$ being the semiconductor vacuum state and $|0\rangle$ the metal ground state, $(H_{\rm m} - \mathbb{E}_0)|0\rangle = 0$.

The excitons (i. e. all bound and extended one-pair eigenstates of the semiconductor hamiltonian, $(H_{\rm sc} - E_{\nu,\mathbf{Q}})B^\dagger_{\nu,\mathbf{Q}}|v\rangle = 0$) are related to the free pairs by

$$B^\dagger_{\nu,\mathbf{Q}} = \sum_{\mathbf{k}} \langle \mathbf{k}|x_\nu\rangle \, a^\dagger_{\mathbf{k}+\alpha_e\mathbf{Q}} \, b^\dagger_{-\mathbf{k}+\alpha_h\mathbf{Q}} \ , \tag{5}$$

$$a^\dagger_{\mathbf{k}_e} \, b^\dagger_{\mathbf{k}_h} = \sum_{\nu} \langle x_\nu|\alpha_h\mathbf{k}_e - \alpha_e\mathbf{k}_h\rangle \, B^\dagger_{\nu,\mathbf{k}_e+\mathbf{k}_h} \ , \tag{6}$$

where $\alpha_e = 1 - \alpha_h = m_e/(m_e + m_h)$, m_e and m_h being the electron and hole masses. Using eq. (6), the semiconductor-photon interaction reads

$$U^\dagger = A\sum_{\mathbf{k}} a^\dagger_{\mathbf{k}+\mathbf{Q}} \, b^\dagger_{-\mathbf{k}} = A\sum_{\nu,\mathbf{k}'} \langle x_\nu|\mathbf{k}'\rangle \, B^\dagger_{\nu,\mathbf{Q}} = A\sum_{\nu} B^\dagger_{\nu,\mathbf{Q}} \langle x_\nu|\mathbf{r} = \mathbf{0}\rangle \ , \tag{7}$$

(if we set the sample volume equal to 1). The response function thus appears as

$$S(\Omega,\mathbf{Q}) = A^2 \sum_{\nu,\nu'} \langle \mathbf{r} = \mathbf{0}|x_{\nu'}\rangle \, S_{\nu'\nu}(\Omega,\mathbf{Q}) \langle x_\nu|\mathbf{r} = \mathbf{0}\rangle \ , \tag{8}$$

$$S_{\nu'\nu}(\Omega,\mathbf{Q}) = \langle i|B_{\nu',\mathbf{Q}} \, \frac{1}{a - H} \, B^\dagger_{\nu,\mathbf{Q}}|i\rangle \ , \qquad\qquad a = \Omega + \mathbb{E}_0 + i\eta \ . \tag{9}$$

In order to calculate $S_{\nu'\nu}(\Omega,\mathbf{Q})$, we can note that

$$\left[H, B^\dagger_{\nu,\mathbf{Q}}\right] = \left[H_{\rm sc}, B^\dagger_{\nu,\mathbf{Q}}\right] + \left[W_{\rm sc-m}, B^\dagger_{\nu,\mathbf{Q}}\right] = (E_{\nu,\mathbf{Q}} B^\dagger_{\nu,\mathbf{Q}} + V^\dagger_{\nu,\mathbf{Q}}) + W^\dagger_{\nu,\mathbf{Q}} \ . \tag{10}$$

The first commutator, calculated in reference (3), shows that $V^\dagger_{\nu,\mathbf{Q}}$ acts on semiconductor electron-hole pairs only so that $V^\dagger_{\nu,\mathbf{Q}}|v\rangle = 0$. Using eqs. (2,5,6), the second commutator gives

$$W^\dagger_{\nu,\mathbf{Q}} = \sum_{\mathbf{q}\neq\mathbf{0},\nu'} \hat{V}_{\nu'\nu}(\mathbf{q}) \, B^\dagger_{\nu',\mathbf{Q}+\mathbf{q}} \, w_{-\mathbf{q}} \ , \tag{11}$$

$$\hat{V}_{\nu'\nu}(\mathbf{q}) = \langle x_{\nu'}|V(\mathbf{q})(e^{i\alpha_h\mathbf{q}.\mathbf{r}} - e^{-i\alpha_e\mathbf{q}.\mathbf{r}})|x_\nu\rangle = \langle x_{\nu'}|\hat{V}(\mathbf{q})|x_\nu\rangle \ . \tag{12}$$

$W^\dagger_{\nu,\mathbf{Q}}$ physically corresponds to excite one exciton from a $(\nu,\mathbf{Q})$ state to a $(\nu',\mathbf{Q}+\mathbf{q})$ state, whereas the metal has one of its electrons excited from $\mathbf{p}$ to $\mathbf{p}-\mathbf{q}$.

It is easy to check that eq. (10) leads to

$$\frac{1}{a - H} \, B^\dagger_{\nu,\mathbf{Q}} = B^\dagger_{\nu,\mathbf{Q}} \, \frac{1}{a - H - E_{\nu,\mathbf{Q}}} + \frac{1}{a - H} \, (V^\dagger_{\nu,\mathbf{Q}} + W^\dagger_{\nu,\mathbf{Q}}) \, \frac{1}{a - H - E_{\nu,\mathbf{Q}}} \ . \tag{13}$$

As $V^\dagger_{\nu,\mathbf{Q}}|v\rangle = 0$, while $W^\dagger_{\nu,\mathbf{Q}}|v\rangle$ writes in terms of $B^\dagger_{\nu',\mathbf{Q}'}$, the iteration of the above equation allows to generate the expansion of $S_{\nu'\nu}(\Omega,\mathbf{Q})$ in the exciton-metal interaction :

$$S_{\nu'\nu}(\Omega,\mathbf{Q}) = \sum_{n=0}^{\infty} S^{(n)}_{\nu'\nu}(\Omega,\mathbf{Q}) \ . \tag{14}$$

The zero order term simply comes from the first term of eq. (13). It reads

$$S^{(0)}_{\nu'\nu}(\Omega,\mathbf{Q}) = \langle i|B_{\nu',\mathbf{Q}}\, B^\dagger_{\nu,\mathbf{Q}}\, \frac{1}{a - H - E_{\nu,\mathbf{Q}}}\, |i\rangle = \delta_{\nu',\nu} G_x(\Omega;\nu,\mathbf{Q}) \ . \tag{15}$$

The first order term appears as

$$S^{(1)}_{\nu'\nu}(\Omega,\mathbf{Q}) = G_x(\Omega;\nu,\mathbf{Q}) \sum_{\mathbf{q}_1\neq 0,\nu_1} \langle i|B_{\nu',\mathbf{Q}} B^\dagger_{\nu_1,\mathbf{Q}+\mathbf{q}_1} \frac{1}{a - H - E_{\nu_1,\mathbf{Q}+\mathbf{q}_1}} \hat{W}_{-\mathbf{q}_1;\nu_1\nu}|i\rangle \ , \tag{16}$$

where we have set $\hat{W}_{-\mathbf{q};\nu'\nu} = \hat{V}_{\nu'\nu}(\mathbf{q}) w_{-\mathbf{q}}$. As $\langle v|B_{\nu',\mathbf{Q}} B^\dagger_{\nu_1,\mathbf{Q}+\mathbf{q}_1}|v\rangle = \delta_{\nu',\nu_1}\delta_{\mathbf{q}_1,\mathbf{0}}$, this first order term is equal to zero.

The second order term reads

$$\begin{aligned} S^{(2)}_{\nu'\nu}(\Omega,\mathbf{Q}) = G_x(\Omega;\nu,\mathbf{Q}) \sum_{\mathbf{q}_2\neq\mathbf{0},\nu_2} \sum_{\mathbf{q}_1\neq\mathbf{0},\nu_1} \langle i|B_{\nu',\mathbf{Q}} B^\dagger_{\nu_2,\mathbf{Q}+\mathbf{q}_1+\mathbf{q}_2} \\ \times \frac{1}{a - H - E_{\nu_2,\mathbf{Q}+\mathbf{q}_1+\mathbf{q}_2}} \hat{W}_{-\mathbf{q}_2;\nu_2\nu_1} \frac{1}{a - H - E_{\nu_1,\mathbf{Q}+\mathbf{q}_1}} \hat{W}_{-\mathbf{q}_1;\nu_1\nu}|i\rangle \ . \end{aligned} \tag{17}$$

The above matrix element can be split into a semiconductor part and a metal part. The first one imposes $\nu_2 = \nu'$ and $\mathbf{q}_1 + \mathbf{q}_2 = 0$, so that

$$S^{(2)}_{\nu'\nu}(\Omega,\mathbf{Q}) = G_x(\Omega;\nu',\mathbf{Q})\ T^{(2)}_{\nu'\nu}(\Omega,\mathbf{Q})\, G_x(\Omega;\nu,\mathbf{Q}) \ , \tag{18}$$

$$T^{(2)}_{\nu'\nu}(\Omega,\mathbf{Q}) = \sum_{\mathbf{q}_1\neq 0,\nu_1} \hat{V}_{\nu'\nu_1}(-\mathbf{q}_1)\langle 0|w_{\mathbf{q}_1} \frac{1}{a - H_{\rm m} - E_{\nu_1,\mathbf{Q}+\mathbf{q}_1}} w_{-\mathbf{q}_1}|0\rangle \hat{V}_{\nu_1\nu}(\mathbf{q}_1) \ . \tag{19}$$

If we neglect Coulomb interaction between metal electrons for simplicity, as in reference (2), $H_{\rm m}\, c^\dagger_{\mathbf{p}-\mathbf{q}} c_{\mathbf{p}}|0\rangle = (\mathbb{E}_0 + \epsilon_{\mathbf{p}-\mathbf{q}} - \epsilon_{\mathbf{p}})\, c^\dagger_{\mathbf{p}-\mathbf{q}} c_{\mathbf{p}}|0\rangle$, $\epsilon_{\mathbf{p}}$ being the metal-electron energy. The matrix element of eq. (19) is thus equal to

$$\sum_{|\mathbf{p}|<k_{\rm F}<|\mathbf{p}-\mathbf{q}_1|} \frac{1}{\Omega - (E_{\nu_1,\mathbf{Q}+\mathbf{q}_1} + \epsilon_{\mathbf{p}-\mathbf{q}_1} - \epsilon_{\mathbf{p}}) + i\eta} \ . \tag{20}$$

We can split it into contributions from the exciton, the metal electron and the metal hole by using the standard trick,

$$\frac{1}{\Omega - a - b + i\eta} = \int \frac{id\omega}{2\pi} \left(\frac{1}{\omega + \Omega - a + i\eta}\right)\left(\frac{1}{-\omega - b + i\eta}\right) \ . \tag{21}$$

Equation (20) then reads

$$\int \frac{id\omega_1}{2\pi}\, G_x(\omega_1+\Omega;\nu_1,\mathbf{Q}+\mathbf{q}_1)\left[-\sum_{\mathbf{p}}\int \frac{id\omega}{2\pi}\, g(\omega,\mathbf{p})\, g(\omega-\omega_1,\mathbf{p}-\mathbf{q}_1)\right] \ , \qquad (22)$$

where $g(\omega,\mathbf{p}) = (\omega - \epsilon_{\mathbf{p}} + i\eta\,\mathrm{sign}(\epsilon_{\mathbf{p}}-\mu))^{-1}$ is the usual metal-electron Green's function, while $G_x(\omega;\nu,\mathbf{Q})$ is the "exciton Green's function" given in eq. (1). This leads to rewrite $T^{(2)}_{\nu'\nu}(\Omega,\mathbf{Q})$ as

$$T^{(2)}_{\nu'\nu}(\Omega,\mathbf{Q}) = \sum_{\mathbf{q}_1\neq 0,\nu_1}\int \frac{id\omega_1}{2\pi}\, B(\omega_1,\mathbf{q}_1)\, \left[\hat{V}_{\nu'\nu_1}(-\mathbf{q}_1)\, G_x(\Omega+\omega_1;\nu_1,\mathbf{Q}+\mathbf{q}_1)\, \hat{V}_{\nu_1\nu}(\mathbf{q}_1)\right] \ , \qquad (23)$$

$B(\omega_1,\mathbf{q_1})$ being the standard "bubble" contribution as given by the bracket of eq. (22). This response function second order term, as well as the zero order term given in eq. (15), correspond to the exciton diagrams shown in fig. (1), with the exciton-metal vertex being $\hat{V}_{\nu'\nu}(\mathbf{q})$.

More generally, the n^{th} order term of $S_{\nu'\nu}(\Omega,\mathbf{Q})$ appears as

$$\begin{aligned} S^{(n)}_{\nu'\nu}(\Omega,\mathbf{Q}) &= G_x(\Omega;\nu',\mathbf{Q})\, G_x(\Omega;\nu,\mathbf{Q}) \\ &\times \left[\sum_{\mathbf{q}_{n-1}\neq\mathbf{0},\nu_{n-1}} \cdots \sum_{\mathbf{q}_1\neq\mathbf{0},\nu_1} \langle 0|\hat{W}_{\mathbf{q}_{n-1}+\cdots+\mathbf{q}_1;\nu'\nu_{n-1}}\, M_{n-1}\, M_{n-2}\cdots M_1|0\rangle\right] \ , \\ M_m &= \frac{1}{a - H_{\mathrm{m}} - E_{\nu_m,\mathbf{Q}+\mathbf{q}_m+\cdots+\mathbf{q}_1}}\, \hat{W}_{-\mathbf{q}_m;\nu_m\nu_{m-1}} \qquad (\nu_0\equiv\nu) \ . \end{aligned} \qquad (24)$$

The bracket corresponds to all the possible ways to start with a $(\nu,\mathbf{Q})$ exciton, to excite it into various $(\nu'',\mathbf{Q}+\mathbf{q}'')$ states while shaking up the metal Fermi sea by $(-\mathbf{q}'')$ and to end with a $(\nu',\mathbf{Q})$ exciton. As an example, the 4^{th} order terms are shown in fig. (2). They are basically of two types : The first term (fig. (2a)) corresponds to excite and recombine one electron-hole pair in the metal Fermi sea, twice. Its contribution to $S^{(4)}_{\nu'\nu}(\Omega,\mathbf{Q})$ is given by

$$G_x(\Omega;\nu',\mathbf{Q})\left[\sum_{\nu_1} T^{(2)}_{\nu'\nu_1}(\Omega,\mathbf{Q})\, G_x(\Omega;\nu_1,\mathbf{Q})\, T^{(2)}_{\nu_1\nu}(\Omega,\mathbf{Q})\right] G_x(\Omega;\nu,\mathbf{Q}) \ . \qquad (25)$$

The other terms of fig. (2) can be formally written as

$$G_x(\Omega;\nu',\mathbf{Q})\, T^{(4)}_{\nu'\nu}(\Omega,\mathbf{Q})\, G_x(\Omega;\nu,\mathbf{Q}) \ , \qquad (26)$$

where $T^{(4)}_{\nu'\nu}(\Omega,\mathbf{Q})$ corresponds to the transfer of the $(\nu,\mathbf{Q})$ exciton into the $(\nu',\mathbf{Q})$ state associated to all possible *connected* excitation processes of the metal Fermi sea with 4 semiconductor-metal interactions.

This shows that the sum of all contributions to $S_{\nu'\nu}(\Omega,\mathbf{Q})$ reads

$$S_{\nu'\nu}(\Omega,\mathbf{Q}) = \delta_{\nu',\nu}\, G_x(\Omega;\nu,\mathbf{Q})$$
$$+G_x(\Omega;\nu',\mathbf{Q})\left[T_{\nu'\nu}(\Omega,\mathbf{Q}) + \sum_{\nu_1} T_{\nu'\nu_1}(\Omega,\mathbf{Q}) G_x(\Omega;\nu_1,\mathbf{Q}) T_{\nu_1\nu}(\Omega,\mathbf{Q}) + \cdots\right] G_x(\Omega;\nu,\mathbf{Q}) \ , \quad (27)$$

where $T_{\nu'\nu}(\Omega,\mathbf{Q})$ corresponds to the transfer of a $(\nu,\mathbf{Q})$ exciton into a $(\nu',\mathbf{Q})$ state associated to the sum of all possible *connected* excitation processes of the metal Fermi sea with two or more semiconductor-metal interactions. This expansion of $S_{\nu'\nu}(\Omega,\mathbf{Q})$ is shown on fig. (3). It corresponds to the expansion of the integral equation shown in fig. (3).

It is in fact possible to rewrite $S_{\nu'\nu}(\Omega,\mathbf{Q})$, as well as $S(\Omega,\mathbf{Q})$, in a quite compact form : For that, we first rewrite the exciton propagator as

$$G_x(\Omega;\nu,\mathbf{Q}) = \langle x_\nu | \frac{1}{\Omega - h_x - \mathcal{E}_{\mathbf{Q}} + i\eta} | x_\nu \rangle \ , \quad (28)$$

where h_x is the exciton relative motion hamiltonian, $(h_x - \varepsilon_\nu)|x_\nu\rangle = 0$. By noting that the second order transfer, given in eq. (23), also reads $T^{(2)}_{\nu'\nu}(\Omega,\mathbf{Q}) = \langle x_{\nu'}|T^{(2)}(\Omega,\mathbf{Q})|x_\nu\rangle$ with

$$T^{(2)}(\Omega,\mathbf{Q}) = \sum_{\mathbf{q}\neq\mathbf{0}} \int \frac{id\omega}{2\pi}\, B(\omega,\mathbf{q})\, \hat{V}(-\mathbf{q})\, \frac{1}{\Omega + \omega - h_x - \mathcal{E}_{\mathbf{Q}+\mathbf{q}} + i\eta}\, \hat{V}(\mathbf{q}) \ , \quad (29)$$

we can, in a similar way, rewrite the higher order transfers as $T_{\nu'\nu}(\Omega,\mathbf{Q}) = \langle x_{\nu'}|T(\Omega,\mathbf{Q})|x_\nu\rangle$. Since $\delta_{\nu',\nu} = \langle x_{\nu'}|x_\nu\rangle$, eq. (27) is nothing but the expansion of

$$S_{\nu'\nu}(\Omega,\mathbf{Q}) = \langle x_{\nu'} | \frac{1}{\Omega - h_x - T(\Omega,\mathbf{Q}) - \mathcal{E}_{\mathbf{Q}} + i\eta} | x_\nu \rangle \ , \quad (30)$$

so that the response function $S(\Omega,\mathbf{Q})$ given in eq. (8) takes the quite compact form,

$$S(\Omega,\mathbf{Q}) = A^2 \langle \mathbf{r}=\mathbf{0} | \frac{1}{\Omega - h_x - T(\Omega,\mathbf{Q}) - \mathcal{E}_{\mathbf{Q}} + i\eta} | \mathbf{r}=\mathbf{0} \rangle \ . \quad (31)$$

The above equation is exactly the eq. (9) of reference (1). The explicit form of this response function was then obtained in terms of the right and left eigenstates $|\hat{x}_\nu\rangle$ and $|\hat{\tilde{x}}_\nu\rangle$ of the non hermitian "hamiltonian" $h_x + T(\Omega,\mathbf{Q})$. As its eigenvalues are complex, the exciton absorption lines in the presence of a 2D metal have now tails.

In conclusion, our commutation technique allows to prove in a quite transparent way that the problem of the exciton absorption spectrum changes induced by the presence of a distant metal, can indeed be solved within exciton diagrams at any order in the semiconductor-metal interaction. These diagrams visualize the fact that a $(\nu,\mathbf{Q})$ exciton

is created by a $\mathbf{Q}$ photon. This $(\nu, \mathbf{Q})$ exciton scatters to a $(\nu_1, \mathbf{Q}+\mathbf{q}_1)$ state and then to a $(\nu_2, \mathbf{Q}+\mathbf{q}_1+\mathbf{q}_2)$ state and so on ... At each $\mathbf{q}_i$ scattering, a $(-\mathbf{q}_i)$ metal electron-metal hole pair is excited. The photocreated exciton must end all these scatterings in a $(\nu', \mathbf{Q})$ state in order to possibly recombine into a $\mathbf{Q}$ photon. On a technical point of view, to each $(\nu', \mathbf{Q}')$ exciton we associate the propagator $G_x(\omega; \nu', \mathbf{Q}')$ given in eq. (1). To each scattering of a $(\nu, \mathbf{Q}; \mathbf{p})$ exciton-metal-electron state into a $(\nu', \mathbf{Q}+\mathbf{q}; \mathbf{p}-\mathbf{q})$ state we associate the exciton-metal vertex $\hat{V}_{\nu'\nu}(\mathbf{q})$, given in eq. (12), and we conserve ω and $\mathbf{q}$ at each vertex, as usual for diagrams.

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FIGURE CAPTIONS

Fig. (1) :

Response function in terms of exciton diagrams, at zero order (a) and at second order (b) in the semiconductor-metal interaction.

To the $(\Omega;\nu,\mathbf{Q})$ exciton, we associate the exciton propagator $G_x(\Omega;\nu,\mathbf{Q})$ given in eq. (1), and to the scattering of a $(\Omega;\nu,\mathbf{Q})$ exciton-$(\omega,\mathbf{p})$ metal electron into a $(\Omega+\omega_1;\nu_1,\mathbf{Q}+\mathbf{q}_1)$ exciton-$(\omega-\omega_1,\mathbf{p}-\mathbf{q}_1)$ metal electron, we associate the exciton-metal vertex $\hat{V}_{\nu_1\nu}(\mathbf{q}_1)$ given in eq. (12).

Fig. (2) :

Exciton diagrams for the response function at 4^{th} order in the semiconductor-metal interaction.

Fig. (3) :

Integral equation verified by $S_{\nu'\nu}(\Omega,\mathbf{Q})$ as given in eq. (27).